Understanding the Role of Trade-Ins in Durable Goods Markets: Theory and Evidence

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The act of trading in a used car as partial payment for a new car resonates with practically all consumers. Such transactions are prevalent in many other durable goods markets, ranging from golf clubs to CT scanners. What roles do trade-ins play in these markets? What motivates the seller to set up a channel to facilitate trade-ins? Intuitively, accepting a trade-in would appear to stimulate demand for the producer’s product, but facilitating the resale of these used goods that substitute for new goods might also increase cannibalization. Although such transactions involve billions of dollars, we know relatively little about this practice from the extant research literature.

This paper develops an analytical model that incorporates key features of real-world durable goods markets: (i) coexistence of new and used goods markets, (ii) consumer heterogeneity with respect to quality sensitivity, (iii) firms that anticipate the cannibalization problem arising from the coexistence of new and used goods, and (iv) lemon problems in resale markets, whereby sellers of used goods are better informed than buyers about the quality of their particular item.

In our analysis, a trade-in policy amounts to an intervention by the firm in the used goods market, which reduces inefficiencies arising from the lemon problem. It motivates owners to purchase new goods and reduces their proclivity to hold on to purchased goods because of the low price the latter would fetch in a lemon market.

We also show that trade-in programs are more valuable for less reliable products, as well as for products that deteriorate more slowly. Our analysis shows that producers in durable goods markets should consider trade-in programs as a matter of routine. Despite cannibalization concerns arising from a more active resale market, a producer’s profits will inevitably rise from introducing trade-ins, given pervasive lemon problems. We test the key predictions of the model about price and volume of trade by assembling a data set of transactions of U.S. automobile consumers and find broad support for our model.

Key words: durable goods; trade-ins; adverse selection; game theory

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purchased goods because of the subpar prices for used goods in these situations. An incentive offered for the purchase of a new good with a trade-in counteracts this tendency.\textsuperscript{3} We show that as the quality uncertainty increases, the magnitude of the trade-in incentive increases, while the volume of used goods trade, the quality of used goods offered, as well as the used good price all decline. On the other hand, as the rate of deterioration increases, the magnitude of the trade-in incentive decreases, while the volume of used goods trade and the quality of used goods offered increase.

Our paper expands on extant work in several ways. First, we use an infinite period model structure and a stationary equilibrium concept accompanying it. This helps us reveal the unique role of trade-ins as a device to mitigate the lemon problem. Second, our model accommodates the cannibalization arising from the coexistence of new and used goods markets, as well as customer heterogeneity and information asymmetry. Finally, we offer empirical evidence about trade-ins by testing a number of refutable predictions from our model.

The remainder of this paper is organized as follows. In §2, we review the extant work on durable goods, with a view to identifying the gaps in our knowledge about trade-in programs and resale markets. Section 3 presents our model, and §4 presents the analysis and results. Section 5 presents our empirical tests of the refutable implications of the model. Section 6 concludes this paper with a discussion of the limitations and possible extensions of the work.

2. Literature
There is a large literature on a number of marketing practices in durable goods markets. These include vertical coordination (Desai et al. 2004), dual channels (Purohit 1997), trade promotions (Bruce et al. 2005), and leasing contracts (Desai and Purohit 1998). However, trade-in practices have received limited attention.

Ackere and Reyneirs (1995) develop a two-period model to show that a discriminatory discount offered to myopic consumers in trade-in transactions will induce more new goods purchases in the second period. However, when the producer can commit to prices intertemporally, or when consumers are forward looking, this rationale for trade-ins vanishes. Adda and Cooper (2000) evaluate a policy intervention similar to trade-ins, whereby French provincial authorities gave owners a one-shot subsidy to scrap their old cars and buy new ones. Their principal finding is that forward-looking consumers shift purchases ahead to take advantage of the discount.

The gaps in our understanding of trade-in policies are seen more clearly when we consider the broader work on durable goods markets. Three gaps are of special significance. First, Coase (1972) pointed out that a durable goods producer who wishes to capitalize on customer heterogeneity in willingness to pay by charging a high price in an initial period followed by a lower price in the following period faces a time commitment problem. Consumers will anticipate this price discrimination strategy and balk at paying the high initial price. In effect, the producer cannot realize his monopoly status. Any contract or institution that enables a producer to commit credibly to prices or quantities over multiple periods solves the problem and re-establishes his monopoly status. A large number of commitment devices have been suggested, including leasing (Desai and Purohit 1999), planned obsolescence (Fudenberg and Tirole 1998), and long-term contracts (Bulow 1982).

Recall the Ackere and Reyneirs (1995) result discussed above where trade-ins play a role only when the producer cannot commit. Clearly, trade-ins in the Ackere and Reyneirs (1995) model are yet another commitment device, indistinguishable from other commitment devices. To uncover any unique properties of trade-ins, we need to work out of a setup where producers can commit intertemporally. Waldman (2003) reaches a similar conclusion from his comprehensive review; namely, future work on durables should proceed under the maintained hypothesis that commitment is achievable by durable goods producers. In our work, we use the concept of a stationary equilibrium (Rust 1985), which renders the commitment problem irrelevant.

A second gap is revealed when we consider the central role of resale markets for durable goods. The coexistence of new and used goods makes these products behave as (imperfect) substitutes, which then raises the specter of cannibalization. A forward-looking firm will internalize the cannibalization arising from current sales on sales in future periods and seek solutions to this problem. The literature has uncovered a large number of solutions to the cannibalization problem, including leasing contracts (Desai and Purohit 1998, Hendel and Lizzeri 1999b), buyback programs (Levinthal and Purohit 1989), planned obsolescence (Fudenberg and Tirole 1998), control over resale markets (Shapiro 1995), and raising the costs of resale transactions (Anderson and Ginsburgh 1994).

\textsuperscript{3} The incentive could take one of two forms. Thus the producer can offer a discount for the new good (referred to hereafter as a “trade-in incentive”) or an “overallowance” for the used good (i.e., a price greater than the market price of the used good). Underlying this equivalence is the fact that a trade-in transaction is a linked transaction—one can arrive at an identical net price for the linked transaction by letting the producer offer a discount on the new good, or a premium in his payment for the used good. We use the term trade-in incentive throughout this paper while acknowledging its equivalence with an overallowance.
Despite the central role of resale markets suggested above, extant work on trade-ins does not include resale markets. In our work, the coexistence of new and used goods is an integral element of the model.

Third, one of the defining characteristics of used goods transactions is missing from the trade-ins literature. Starting with Akerlof (1970), the information asymmetry between buyers and sellers of used goods has been viewed as a central element of many markets. The incumbent owner cannot communicate the quality of her used good credibly to the buyer, leading to only lemons being offered for sale, which then shuts down the used goods market. Recent modeling (e.g., Hendel and Lizzeri 1999a) has shown that used goods markets do not shut down completely in the face of the lemon problem, but the volume of trade (VOT) does go down. Our work explicitly incorporates the lemon problem into the model. In that sense, our paper is closely related to prior modeling literature that has examined the effects of asymmetric information on market outcomes (Balachander and Srinivasan 1994, Tellis and Wernerfelt 1987) and rational firms’ response to it. Trade-ins, we suggest, help ameliorate problems caused by asymmetric information.

3. Model

Consumers. We assume a fixed number of infinitely lived, utility-maximizing consumers who consume at most one unit of a durable good per period. We model these heterogeneous consumers with a vertical differentiation model (Mussa and Rosen 1978); i.e., consumers have identical preferences for quality, but differ in their willingness to pay for it. For tractability, we assume a uniform distribution of the valuation of quality, $\theta$, over the set $[\theta, \bar{\theta}]$, and normalize $\bar{\theta} = 0$ and $\bar{\theta} = 1$. The consumer’s per period linear indirect utility function (ignoring income effects) is $U(\theta, \eta) = \theta \eta - p$, where $\eta$ is per period quality of the good, and $p$ is the price of the good.

Product. The product lasts two periods and is denoted “new” in the first period and “old” in the second. A new product provides a quality of $\nu$, which represents the total utility from using the product, including functional, aesthetic, and/or hedonic utility. The new good deteriorates to a high-quality used good (a peach) with probability $\alpha$, or to a low-quality used good (a lemon) with probability $(1 - \alpha)$. After period 2, the good becomes useless and is scrapped costlessly.

Without loss of generality, we assume that a peach provides a quality of $(1 + s)$, while a lemon provides a quality of $(1 - s)$. Furthermore, we assume $\alpha$ equals $\frac{5}{6}$; i.e., a new good has an equal probability of becoming a peach or a lemon in the second period. The parameter $s$ thus indexes the extent of quality uncertainty about used goods. Throughout this paper, we assume that $0 < s < 1$ (Assumption A1), to rule out the implausible case of used goods with negative utility. We also assume that $s < (4/5) (\nu - 1)$ (Assumption A2), so that $s$ is not “too large.” This guarantees (i) a positive flow of goods every period, (ii) the new good deteriorates after one period, (iii) the used good market is active, and (iv) positive prices.

Producer. An infinitely lived profit-maximizing producer chooses the price of his new good, as well as whether to offer a trade-in program (a trade-in program refers to his decision to accept a used good as partial payment for a new good). The marginal cost of production is normalized to zero without loss of generality.

Adverse Selection. The quality level of the used good is private information to the seller. Buyers form expectations about the average quality of these goods, but they are uncertain of the quality of a particular item. This degree of quality uncertainty is captured by $s$. The higher the value of $s$, the more uncertain the buyers of a used good are about its quality. Note that it is not crucial that sellers know the quality of the used good with certainty; all we need is that sellers have a relative informational advantage over buyers, with $s$ being a proxy for this advantage.

For ease of reference, Table 1 provides a summary of the notation used.

4. Analysis

4.1. Characterizing Consumer Behavior and the Stationary Equilibrium

In a general setup, in every period, each consumer makes a consumption decision based on the prices prevailing in the market, as well as her own consumption decision in the previous period. In effect, both consumers and the firm solve a dynamic maximization problem. Consumers take new and used prices as a given and decide their consumption decision at the beginning of each period, while the firm

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4 Formally, a lemon has a quality level lower than the level expected by the buyer.

5 The origins of these terms are obscure, but they are commonly used in the United States and other English-speaking countries. For instance, http://www.peachorlemon.co.uk provides automobile reviews of UK cars, with five peaches and five lemons representing the best and worst used cars, respectively.

6 This assumption is made solely for tractability and exposition; our results are robust to an entire range of $\alpha$. See §4.3.4 for further discussion.

7 For most of our results to go through, we only need a much weaker assumption than A2; namely, $\nu > (1 + s)$, which implies that a new good deteriorates for sure after one period.
sets the new good price on the basis of anticipated consumer behavior. Used goods prices are determined competitively through market clearing. For expositional purposes, we refer to a consumer’s decision in a particular period as an action, while her action in the previous period is referred to as her state in the current period.

In any period, the consumer can take one of three actions—she can (i) buy a new good, (ii) buy a used good, or (iii) not buy a good. Action (iii), in turn, could arise in two cases, distinguished by the consumer’s state. Thus she could have bought a new good in the last period and decided to hold on to the good this period (the good could have turned out to be a peach or a lemon), or she may not have bought a good at all in the last period. Formally, we can represent the action that a consumer can take in any particular period as the following five-dimensional binary vector:

\[ b^t(\theta) = [n^t(\theta), h^t_p(\theta), h^t_l(\theta), u^t(\theta), i^t(\theta)], \]

where

\[ n^t(\theta) = 1 \text{ refers to a purchase of a new good by type } \theta \text{ at time } t. \]
\[ h^t_p(\theta) = 1 \text{ refers to the holding of a (good that was revealed to be a peach by type } \theta \text{ at time } t. \]
\[ h^t_l(\theta) = 1 \text{ refers to holding a (good that was revealed to be a lemon by type } \theta \text{ at time } t. \]
\[ u^t(\theta) = 1 \text{ refers to purchase of a used good by type } \theta \text{ at time } t. \]
\[ i^t(\theta) = 1 \text{ refers to type } \theta \text{ not using any good at time } t. \]

Note that a nonzero value of \( h^t_p(\theta) \) or \( h^t_l(\theta) \) is possible only when the state of the consumer is \( n^{t-1}(\theta) = 1 \); i.e., holding in the current period \( t \) necessarily implies that a new good was purchased in the previous round \( (t-1) \). Additionally, we impose the following constraint: \( n^t(\theta) + h^t_p(\theta) + h^t_l(\theta) + u^t(\theta) + i^t(\theta) = 1 \), which ensures consumers have at most one good during any period.

Let \( H_\theta[b^{t-1}(\theta), b^t(\theta), p^t] \) be the payoff for consumer \( \theta \) given state \( b^{t-1}(\theta) \), action \( b^t(\theta) \), and price vector \( p^t \) (new and used prices). The following Bellman equation outlines the action that a consumer would choose to maximize her utility:

\[
V^t(\theta)[b^{t-1}(\theta), p^t] = \max_{b^t(\theta)} \left[ H_\theta[b^{t-1}(\theta), b^t(\theta), p^t] + \delta V^{t+1}(\theta)[b^t(\theta), p^{t+1}] \right].
\] (1)

Note that in (1), the action \( b^t(\theta) \) in period \( t \) becomes a state variable in period \( t + 1 \). Given the consumer behavior outlined above, the producer determines a new good price each period to maximize his profits. A stationary equilibrium in the above setting is one where prices as well as aggregate consumer behavior remain constant across time. The stationary equilibrium is essentially a fixed point in strategy space that players converge to. Hence, this concept is well suited to understanding the long-run behavior of firms and consumers. The existence of such an equilibrium in this setting has been formally shown in Rust (1985) and Konishi and Sandfort (2002).

Before we proceed to characterize the stationary equilibrium in this setup, we would like to comment on its appropriateness for our research question. To be sure, a number of papers have used this concept to examine similar issues in a durable goods setting, e.g., the impact of adverse selection on prices (Hendel and Lizzier 1999a), trading patterns across vintages (Stolyarov 2002), and the presence of simultaneous leasing and selling in durable markets (Huang et al. 2001, Gavazza 2005, Gilligan 2004). Nevertheless, just like any other equilibrium concept, stationary equilibrium has some limitations. An obvious alternative is a subgame-perfect Nash equilibrium. However, while this alternative would capture nonstationary changes between one period and the next, it suffers from an “end-of-the-world” feature that crucially influences any results one finds; e.g., it necessarily implies a commitment problem that trade-ins would then help to solve. Our aim in this paper is to focus on the unique features of trade-ins other than as a solution to the commitment problem while accommodating key features of durable goods markets, such as the coexistence of new and used goods, customer heterogeneity, and cannibalization arising out of new and used goods competing as differentiated goods.

The characterization of the stationary equilibrium proceeds as follows.\(^9\) In the first step, we search for

<table>
<thead>
<tr>
<th>Table 1</th>
<th>List of Symbols</th>
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<tbody>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Consumer’s valuation for quality</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Per period service flow of a new good</td>
</tr>
<tr>
<td>( s )</td>
<td>Spread of used good quality</td>
</tr>
<tr>
<td>( a )</td>
<td>Probability of realization of a peach</td>
</tr>
<tr>
<td>( w )</td>
<td>Expected used good quality</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Discount factor</td>
</tr>
<tr>
<td>( p )</td>
<td>Price of a new good without a trade-in</td>
</tr>
<tr>
<td>( p_r )</td>
<td>Price of a new good with a trade-in</td>
</tr>
<tr>
<td>( p_\nu )</td>
<td>Price of a good that is a used good</td>
</tr>
<tr>
<td>( y )</td>
<td>New good flow per period</td>
</tr>
<tr>
<td>( h_\theta )</td>
<td>Marginal compulsive buyer</td>
</tr>
<tr>
<td>( h_\delta )</td>
<td>Marginal strategic holder</td>
</tr>
<tr>
<td>( i_\theta )</td>
<td>Marginal cheapskate</td>
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\(^{9}\) In stationary equilibrium, the time dependency in the value function outlined in Equation (1) drops out. After the completion of each optimal pattern of behavior, the consumer returns to the start of the same cycle again. This greatly simplifies the calculation of the closed-form solutions of the equilibrium outcomes.
feasible patterns of consumer behavior. Because a good lasts for two periods, we only need to look at all the possible actions of a consumer at two consecutive periods of time. We narrow these down to feasible patterns of consumer behavior that are consistent, incentive compatible, and individually rational. In the second step, we classify the consumer segments that follow the behaviors just outlined. Table 2 shows the payoff matrix for a consumer of type \(i\).

Table 2 Payoff Matrix for a Consumer of Type \(i\)

<table>
<thead>
<tr>
<th>Binary vector</th>
<th>(n^{-1}(\theta))</th>
<th>(h^{-1}(\theta))</th>
<th>(l^{-1}(\theta))</th>
<th>(u^{-1}(\theta))</th>
<th>(i^{-1}(\theta))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(n'(\theta))</td>
<td>(n' - \rho + \rho)</td>
<td>(n' - \rho)</td>
<td>(n' - \rho)</td>
<td>(n' - \rho)</td>
<td>(n' - \rho)</td>
</tr>
<tr>
<td>(h'_{1}(\theta))</td>
<td>(1 + s\theta)</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>(h'_{2}(\theta))</td>
<td>(1 - s\theta)</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>(u'(\theta))</td>
<td>(w\theta)</td>
<td>(w\theta - p_u)</td>
<td>(w\theta - p_u)</td>
<td>(w\theta - p_u)</td>
<td>(w\theta - p_u)</td>
</tr>
<tr>
<td>(i'(\theta))</td>
<td>(p_u)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Notes. \(n'(\theta) = 1\) refers to a purchase of a new good by type \(\theta\) at time \(t\). \(h'_{1}(\theta) = 1\) refers to the holding of a (good that was revealed to be a) peach by type \(\theta\) at time \(t\). \(h'_{2}(\theta) = 1\) refers to holding a (good that was revealed to be a) lemon by type \(\theta\) at time \(t\). \(u'(\theta) = 1\) refers to purchase of a used good by type \(\theta\) at time \(t\). \(i'(\theta) = 1\) refers to type \(\theta\) not buying any used good at time \(t\). \(\rho\) is the new good price. \(p_u\) is the used good price. \(w\) is the expected quality of used goods. \(s\) is the quality uncertainty. \(\nu\) is the deterioration.

4.2. Markets with Adverse Selection but Without Trade-Ins

We begin by characterizing markets with adverse selection issues, but without trade-ins. Recall that \(s\) represents the spread of used good quality—as \(s\) increases, buyers are more uncertain about quality. Potential used goods buyers form an expectation about the quality of the used good offered for sale. As outlined in the previous section, there are only four patterns of behavior to consider within this market. We now write down the value functions of each

Proofs of all the lemmas and propositions in this paper are provided in the appendix. Other technical details are available in an accompanying Technical Appendix, available at http://mktsci.pubs.informs.org.

Note the difference between the terms quality uncertainty and adverse selection. One could have a market with the former and not the latter. Thus, one could have quality uncertainty revealed at the end of the first period to every consumer. At the other extreme, and less realistically for our context, one could have the quality realization revealed to no consumer. In either case, there is clearly no adverse selection, because there is no asymmetry of information. In what follows, we speak only to the case where adverse selection is present, i.e., quality uncertainty coupled with asymmetric information.

We prove this result formally in the Technical Appendix, available at http://mktsci.pubs.informs.org (see Lemma T01).

Note that the payoff for each action is net of price paid and other costs, including the cognitive costs of making decisions on whether to trade in (which have been normalized to zero).
of these segments, as a prelude to characterizing the equilibrium.

Segment 1 (Compulsive Buyers): These consumers buy a new good every period and sell it after one period, which yields them a discounted utility of

$$V^b(\theta) = \frac{v\theta - p + \delta p_u + V^c(\theta)}{1 - \delta},$$

which can be expressed as

$$V^b(\theta) = \frac{v\theta - p + \delta p_u}{1 - \delta},$$

where $V$ represents the consumer’s value function (with superscript $b$ indicating the compulsive buyer segment), $p$ is the new good price, $p_u$ is the used good price, and $\delta$ is the discount rate. Segment 2 (Strategic Holders): These consumers exhibit state-dependent behavior, i.e., they buy a new good, and hold if the new good deteriorates into a peach, but sell if it deteriorates into a lemon. The discounted utility of these consumers is

$$V^h(\theta) = \frac{v\theta - p + (1 - \alpha)\delta p_u + V^c(\theta)}{1 - (1 - \alpha)\delta - \alpha\delta^2} + (\alpha)\delta[(1 + s)\theta + \delta V^c(\theta)],$$

where the superscript $h$ indicates the strategic holder segment. Simplifying, we get

$$V^h(\theta) = \frac{v\theta - p + (1 - \alpha)\delta p_u + (\delta)(\alpha)(1 + s)\theta}{1 - (1 - \alpha)\delta - \alpha\delta^2}. \quad (3)$$

Segment 3 (Cheapskates): These consumers buy used goods every period. Their discounted utility is

$$V^c(\theta) = \frac{w\theta - p_u}{1 - \delta},$$

where the superscript $c$ indicates the cheapskate segment and $w$ is the consumer’s expectation about the quality of used goods in the market. Segment 4 (Nonbuyers): These consumers do not buy goods in any period and get zero utility.

4.2.1. Solution Strategy. The producer sets the price of the new good recognizing the consumer behavior patterns delineated above. The price determines the size of each of the segments, which, in turn, determines the quantity of the new good demanded as well as the quality of the used good supplied. Our calculation of the equilibrium solution consists of the following steps:

Step 1. Taking new and used good prices as a given, we calculate the marginal consumer for each of the segments described above.

Step 2. Using the market clearing condition and the fact that new good flow per period is constant in a stationary equilibrium, we calculate the new and used good prices in terms of expected used good quality and new good flow per period.

Step 3. We use segment sizes to determine the used good quality being supplied, which, in equilibrium, must equal the expected used good quality.

Step 4. We calculate the new good price that optimizes per period profits for the producer.

4.2.2. Calculating Segment Boundaries. The marginal consumer ($\theta_1$) at the boundary between the compulsive buyer and strategic holder segments is found by equating (2) and (3). This gives

$$\theta_1 = \frac{p - (1 + \delta) p_u}{v - (1 + s)}. \quad (5)$$

The marginal consumer ($\theta_2$) at the boundary between the strategic holder and cheapskate segments is found by equating (3) and (4):

$$\theta_2 = \frac{p - (1 + \delta) p_u}{(1 + s)\alpha\delta + v - (1 + \alpha\delta) w}. \quad (6)$$

Finally, the marginal consumer ($\theta_3$) at the boundary between the cheapskate segment and the nonbuyer segment is found by equating (4) to zero:

$$\theta_3 = \frac{p_u}{w}. \quad (7)$$

Observe that our expressions for prices involve segment boundaries, which are themselves derived endogenously. In what follows, we derive prices and segment boundaries completely in terms of exogenous parameters, along with equilibrium profits and the optimal quantity of new goods produced.

4.2.3. Deriving Equilibrium Prices and Quantities. Denote $y$ as the producer’s choice of new good output per period. In the appendix (Lemma TA1), we show that in stationary equilibrium, $1/(1 + a)$ of the strategic holder segment buys new goods every period, while $\alpha/(1 + a)$ holds. Using this result, we can write

$$\frac{1 - \theta_1}{compulsive \ \ buyers} + \frac{1}{strategic \ \ holders} \left(\frac{\theta_1 - \theta_2}{cheapskates}\right) = y. \quad (8)$$

Market clearing requires that

$$\left(\begin{array}{c} 1 - \theta_1 \\ \theta_1 - \theta_2 \end{array}\right) + \left(\begin{array}{c} 1/a \\ 1 + a \end{array}\right) \left(\begin{array}{c} \theta_1 - \theta_2 \\ \theta_2 - \theta_3 \end{array}\right) = (y - \theta_3). \quad (9)$$

4.2.4. Calculating the Used Quality. Recall that compulsive buyers always sell their purchased good regardless of its quality realization, while strategic holders sell only if they realize a lemon. The average used good quality supplied by compulsive buyers is
[\alpha(1+s)+(1-\alpha)(1-s)], while strategic holders supply an average quality of \((1-s)\). In a rational expectations framework, the expected used quality equals the used quality supplied:

\[
\text{used quality supplied} = \frac{(\theta_1)(\alpha(1+s)+(1-\alpha)(1-s)) + ((1-\alpha)/(1+\alpha))(\theta_1-\theta_2)(1-s)}{(\theta_1-\theta_2) + ((1-\alpha)/(1+\alpha))(\theta_1-\theta_2)}
\]

Using Equations (5)–(7), and (10), we can solve (8) and (9) simultaneously to get \(p\) and \(p_u\). The final step involves the calculation of the optimal profit and per period new good output. The results are not reproduced here to save space.

4.3. Markets with Adverse Selection and Trade-Ins

We now turn to an analysis of markets with adverse selection problems, but where the producer can offer trade-ins; i.e., consumers can bring in their used good as partial payment for a new good. Denote \(p_T\) as the new good price for buyers who trade in, and \(p\) as the new good price for buyers who do not trade in. The producer resells the used good at its resale market price, \(p_u\). We can view these prices in three ways. In one approach, producers offer two new good prices; \(p_T\) the new good price for buyers who trade in, and \(p\) the new good price for buyers who do not trade in, with the traded-in good being paid for at its resale market price of \(p_u\). Alternatively, producers offer a single new good price, \(p\), but offer a payment \((p-p_T+p_u)\) for the traded-in good.\(^{14}\)

In either case, trade-ins exist only if \(p-p_T+p_u \geq p_u\). Otherwise, all consumers would sell directly into the resale market. Put differently, a consumer who trades in needs to be charged a lower effective price (including the value of the traded-in good) than a buyer who does not trade in.\(^{15}\) As before, we delineate the four possible patterns of consumer behavior.

Segment 1 (Cheapskates): Because the trade-in price is lower than the nontrade-in price, these consumers buy a new good every period through a trade-in, regardless of their realization of used good quality. Their discounted utility is

\[
V^c(\theta) = \frac{w\theta - p_T + \delta p_u}{1 - \delta - \alpha\delta^2}.
\]

Segment 2 (Strategic Holders): These consumers hold their peaches but trade in their lemons for new goods. Their discounted utility is

\[
V^b(\theta) = \nu\theta - ((1-\alpha)p_T + ap) + (1-\alpha)\delta(p_u + V^h(\theta)).
\]

Simplifying, we get

\[
V^b(\theta) = \frac{w\theta - p_T + \delta p_u}{1 - \delta - \alpha\delta^2}.
\]

The utility of these consumers depends on the new good quality, used good price, and size of \(s\). Larger values of \(s\) increase the utility from holding peaches. Intuitively, buying a new good that deteriorates stochastically includes the option of holding a peach. Thus the higher the \(s\), the higher the value of this option. The new good price is effectively \((1-\alpha)p_T + ap\) because these consumers trade in lemons (and pay a price \(p_T\) for the new good) but hold peaches (subsequently, paying \(p\) for a new good), and these two events have probabilities \((1-\alpha)\) and \(\alpha\), respectively.

Segment 3 (Cheapskates): These consumers buy a used good every period. Their discounted utility is

\[
V^c(\theta) = \frac{w\theta - p_u}{1 - \delta}.
\]

Segment 4 (Nonbuyers): These consumers do not buy in any period, and get zero utility.

4.3.1. Calculating Segment Boundaries. Using (11)–(13) and performing calculations similar to the ones in §4.2.2, we can immediately derive the marginal consumer (\(\theta_1\)) at the boundary between the compulsive buyer and strategic holder, the marginal consumer (\(\theta_2\)) located at the boundary between the strategic holder and cheapskate segments, and the marginal consumer (\(\theta_3\)) at the boundary between the cheapskate and nonbuyer segments;\(^{16}\) respectively, as

\[
\theta_1 = \frac{p + (1+\delta)(-p_T + \delta p_u)}{\delta(1+s-\nu)},
\]

\[
\theta_2 = \frac{(1-\alpha)p_T + ap - (1+\delta)p_u}{(1+s)\alpha\delta + \nu - (1+\alpha\delta)w},
\]

\[
\theta_3 = p_u/w.
\]

\(^{14}\) A brief comment is in order about the used good market. The producer pays the prevailing used good price (determined through market clearing) for the good traded in. He then sells back the traded-in good to the secondary market at the prevailing used good price. As such, goods traded in to the producer are the only source of used goods in our model.

\(^{15}\) We do not impose this condition—it emerges naturally in equilibrium (see Proposition TO1 in the Technical Appendix, available at http://mktsci.pubs.informs.org, for a formal proof).

\(^{16}\) Note that no particular individual can uniquely be identified ex ante as always holding or always trading in. In equilibrium, however, the segment’s behavior in aggregate is constant over time.
### 4.3.2. Deriving Equilibrium Prices and Quantities

Let $y$ be the optimal flow of new goods every period (determined by the producer’s optimization problem). Again, using the result (Lemma TA1) that in stationary equilibrium, $1/(1 + \alpha)$ of the strategic holder segment buys new goods every period, while $\alpha/(1 + \alpha)$ holds, we have

$$\frac{(1 - \theta_1) + (1/(1 + \alpha)) (\theta_1 - \theta_3)}{1/(1 + \alpha) (\theta_1 - \theta_3)} = y. \quad (17)$$

Market clearing requires that

$$\frac{(1 - \theta_1) + \left(\frac{(1 - \alpha)}{(1 + \alpha)}\right) (\theta_1 - \theta_3)}{\frac{(1 - \alpha)}{(1 + \alpha)} (\theta_1 - \theta_3)} = (\theta_2 - \theta_3). \quad (18)$$

Using Equations (14)–(16), and solving (17) and (18) simultaneously yields expressions for new and used good prices. As before, we solve for used good quality by equating buyers’ expected used good quality with used good quality supplied. Given these expressions for prices and used good quality, we solve the producer’s per period profits:

$$\arg \max_{p_T, p \in \mathbb{R}_+^2} (1 - \theta_1) p_T + \left(\frac{(1 - \alpha)}{(1 + \alpha)}\right) (\theta_1 - \theta_2) p_T$$

$$+ \left(\frac{\alpha}{1 + \alpha}\right) (\theta_1 - \theta_2) p. \quad (19)$$

Maximizing these profits with respect to $p_T$ and $p$ leads to the equilibrium solutions in Table 3.\footnote{Although the results in Table 3 are for the case where $\delta \to 1$, there is no loss of generality. Also, while the final expressions are calculated at $\alpha = \frac{1}{2}$ for tractability and exposition, our results are robust to a wide range of $\alpha$.}

### 4.3.3. Intuition and Comparative Statics

Before we move on to a more detailed examination of the propositions flowing from our model, observe that the producer practices differential pricing, in that the new good price charged to a consumer who buys with a trade-in is different from that charged to a consumer who buys without a trade-in (in Table 3, $p_T$ is not equal to $p$). In essence, the producer gives an incentive to consumers who buy a new good with a trade-in. We refer to this incentive as a “trade-in incentive.”

It is crucial to highlight that we would not observe differential prices in the absence of adverse selection (see Corollary 1 in the Technical Appendix, available at http://mktsci.pub.informs.org). This reiterates the point that in our model, trade-ins serve as a mechanism to ameliorate the adverse selection problem.

\footnote{As an aside, note that the existence of trade-ins in equilibrium implies that the producer’s profits are higher when offering trade-ins than otherwise (see Proposition T1A in the appendix).}
We consider the intuition behind this differential pricing by comparing worlds with and without trade-ins. In each case, consider the case of positive quality uncertainty (i.e., $s > 0$). This uncertainty, coupled with the fact that quality is revealed only through use, and only to the user, gives rise to the adverse selection (lemon) problem. How exactly does adverse selection impact consumer behavior in our model, and how do trade-ins modify this behavior? Understanding this is the key to comprehending the role of trade-ins.

First, note that as quality uncertainty increases, the peach is a better peach and the lemon is a worse lemon. Buying the new good confers an option value on the buyer—the more alluring the peach, the greater the value of the option. This tempts consumers to hold—recall that, in our model, the strategic holder segment comprises consumers who hold if the realization is a peach, and sell if it is a lemon. This causes the segment of strategic holders to expand. The expansion comes at the expense of both the compulsive buyers (who buy new every period) and the cheapskates (who buy used every period). There are now fewer compulsive buyers because the utility from a better peach is enough to tip some consumers over to the holder segment. There are also fewer cheapskates because the quality of used goods being offered for sale is lower (the lemons are worse).

All of the above are true both with and without trade-ins. Trade-ins help ameliorate the problem caused by adverse selection, by providing an extra wedge to help shrink the size of the strategic holder segment. This is accomplished by charging a relatively higher price to consumers who buy new without trading in compared to consumers who buy new with a trade-in. This has the effect of discouraging consumers from holding on to their used good—the trade-in regime always has a smaller strategic holder segment than the regime without trade-ins. As a counterpart to this, the compulsive buyer and cheapskate segments are larger with trade-ins than without.

In summary, trade-ins work by squeezing the strategic holder segment from both sides, i.e., by expanding the size of both the compulsive buyer and the cheapskate segments. Figure 1 summarizes the above discussion by illustrating segment sizes under both trade-in and no trade-in regimes.

So far, we have derived the producer’s optimal prices and quantities and discussed consumer behavior in the presence and absence of trade-ins. We now turn to characterizing the effect of exogenous parameters on equilibrium outcomes. We are principally interested in two exogenous parameters—the quality uncertainty ($s$) and the deterioration ($\nu$). Similarly, we are principally interested in two outcomes—the magnitude of the trade-in incentive and the volume of used good trade. We choose these outcomes because they are amenable to empirical testing. The combination of these two parameters and two outcomes yields four testable propositions.

The Impact of Quality Uncertainty ($s$) on Trade-In Incentive: As discussed earlier, a larger $s$ increases the option value of holding. Trade-ins help discourage this behavior, by using the trade-in incentive as a wedge to separate consumers who trade-in from those who do not. Clearly, as quality uncertainty increases, the producer has to do more to price discriminate; i.e., he has to offer a larger incentive. The proposition below states this formally.

**Proposition 1.** As the quality uncertainty of used goods increases, the trade-in incentive increases; i.e., $\frac{\partial(p - p_T)}{\partial s} > 0$.

The Impact of Quality Uncertainty ($s$) on Volume of Used Good Trade: Adverse selection is essentially a friction in the marketplace, leading to a reduction in used good transactions. In Akerlof’s (1970) original model, the friction is enough to shut the market down entirely. In our case, the market still functions, and it is of interest to see what factors impact the number of transactions of used goods that take place and how. Holding behavior would clearly reduce the VOT. As outlined earlier, a higher $s$ leads to a larger segment of potential holders. While all compulsive buyers sell back used goods irrespective of quality realization, potential holders keep the peaches. Hence, a larger potential holder segment would imply more
consumers holding on to their cars, thus decreasing the VOT in the used good market. Formally:

**Proposition 2.** As the quality uncertainty increases, the VOT in the used good market decreases; i.e., $\frac{\partial \text{VOT}}{\partial s} < 0$.

**Impact of Deterioration ($\nu$) on Trade-In Incentive:** Recall that a new good provides a first-period quality of $\nu > 1$, while a used good provides a second-period quality of 1, and a quality of 0 thereafter. Thus, a larger $\nu$ represents a steeper deterioration rate. A higher $\nu$ makes a used good a poorer substitute for a new good. This has a direct impact on the incentive to hold, because the option value of holding decreases. In terms of segment size, the strategic holder segment shrinks, because even the prospect of a peach is not alluring enough. Figure 2, which is derived from the results shown in Table 3, with suitable values from 0.1 to 0.8 and $\nu$ taking values from 0.2 to 0.8 in increments of 0.1. We find that all four propositions derived above are robust to these different values. Some examples of these numerical simulations, along with details on their calculation, are provided in the Technical Appendix, available at http://mktsci.pubs.informs.org (see §TO4).

**4.3.4. Additional Analyses.**

**Robustness of Results for Different Values of $\alpha$:** The results described in the previous subsection were for the specific value of $\alpha = \frac{1}{2}$. To check the robustness of our results, we ran a series of simulations for a wide range of values of $\alpha$. Specifically, we obtain numerical solutions for a series of values of $\alpha$ going from 0.2 to 0.8 in increments of 0.1. We find that all four propositions derived above are robust to these different values. Some examples of these numerical simulations, along with details on their calculation, are provided in the Technical Appendix, available at http://mktsci.pubs.informs.org (see §TO5).

**Robustness of Results for Asymmetrical Peaches and Lemons:** In the analysis so far, we have assumed that the quality of a peach is $(1+s)$ while the quality of a lemon is $(1-s)$. To check the robustness of our results for asymmetrical peaches and lemons, i.e., a peach with quality $(1+s_1)$ and a lemon with quality $(1-s_2)$, we ran a series of numerical simulations with $s_1$ taking values from 0.1 to 0.8 and $s_2$ taking values from 0.1 to 0.8 in steps of 0.1. We find that all four propositions derived above are robust to these different values. Some examples of these numerical simulations, along with details on their calculation, are provided in the Technical Appendix, available at http://mktsci.pubs.informs.org (see §TO5).

**5. Empirical Study**

**5.1. Empirical Research Context and Hypotheses Tested**

We conduct an exploratory empirical study that aims to test some of the main propositions flowing out of our model. We use the automobile market as our empirical context. Although it is true that it is very hard to find a field setting that maps on perfectly to
a stylized analytical model, the context we use seems appropriate in that automobiles are durable products that deteriorate with age and are characterized by steady improvements rather than abrupt changes in technology. There are active resale markets for used automobiles and virtually all producers accept trade-ins. That said, it is useful to point out some differences between our analytical setup and the empirical context. For example, (i) in our model, the firm is a monopolist, while the empirical context features an oligopoly; (ii) we do not consider intermediaries in our model, while the car market has dealers who are distinct from manufacturers; (iii) a good in our model lasts only for two periods, while cars can last multiple periods; and (iv) in our model, the only source of used goods for the secondary market is goods traded in to the manufacturer. In actual automobile markets, one would have separate “private party sales” and “dealer sales” of used goods.

There are an enormous variety of vehicles in the marketplace, so we need to define our unit of analysis carefully. Following past work (e.g., Goldberg 1996), we define our unit of analysis as a make-model combination (e.g., Ford Taurus, Toyota Camry). We use a four-year window (1999–2002) for our observations.

5.2. Empirical Specification

Testing Propositions Related to the Trade-In Incentive: Proposition 1 implies \( \Delta(p - p_t)/\Delta s > 0 \), while Proposition 3 implies \( \Delta(p - p_t)/\Delta p < 0 \). We test both these propositions using the following econometric specification:

\[
NetPricePaid_{ij} = \beta_0 + \beta_1 \text{TradeIn}_{ij} + \beta_2 \text{Reliability}_{ij} + \beta_3 \text{Deterioration}_{ij} + \beta_4 (\text{TradeIn}_{ij} \times \text{Reliability}_{ij}) + \beta_5 (\text{TradeIn}_{ij} \times \text{Deterioration}_{ij}) + \eta H_t + \lambda Z_j + \delta X_{ij} + \xi Q_{ij} + \epsilon_{ij},
\]

where the subscripts refer to household \( i \), new vehicle \( j \) in year \( t \). The parameter \( \beta_{0ij} \) captures make-model effects, and \( \text{TradeIn}_{ij} \) is a dummy variable indicating whether the new purchase by household \( i \) involved a trade-in.\(^{19}\)

To see the link between the analytical propositions and the econometric specification, it is useful to rewrite the net price paid for a new automobile by a buyer without trade-in as

\[
\text{Without Trade-In Net Price Paid} = \beta_2 (\text{Reliability}) + \beta_3 (\text{Deterioration}) + \Delta,
\]

where \( \Delta \) is an omnibus term representing the rest of the estimation equation.

Similarly, the net price for consumers purchasing with a trade-in is given by

\[
\text{With Trade-In Net Price Paid} = \beta_1 (\text{Trade-In Dummy}) + \beta_2 (\text{Reliability}) + \beta_3 (\text{Deterioration}) + \beta_4 (\text{Trade-In Dummy} \times \text{Reliability}) + \beta_5 (\text{Trade-In Dummy} \times \text{Deterioration}) + \Delta.
\]

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\]

where the subscripts refer to household \( i \), new vehicle \( j \) in year \( t \). The parameter \( \beta_{0ij} \) captures make-model effects, and \( \text{TradeIn}_{ij} \) is a dummy variable indicating whether the new purchase by household \( i \) involved a trade-in.\(^{19}\)

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\[
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\]

It follows from Equation (23) that Proposition 1 would be supported by a positive \( \beta_4 \), implying that an increase in reliability (equivalently, a decrease in quality uncertainty) is associated with a decrease in the trade-in incentive. Similarly, Proposition 3 would be supported by a positive \( \beta_5 \), implying that an increase in deterioration is associated with a decrease in the trade-in incentive.

A brief word on the other variables included in the specification: the vector of household characteristics, \( H \), is included to control for price effects arising from bargaining differences across households and possible discriminatory effects facing minority and female heads of households that have been documented in previous studies (e.g., Ayers and Siegelman 1995). The vector of vehicle characteristics, \( Z \), is included to control for price differences arising from the differences in trim levels within a make-model. The vector of year dummies, \( X \), is included to control for year-specific shocks that impact the automobile industry, and the vector of quarter dummies, \( Q \), is included to control for seasonal variation in prices within the quarters of a year and the model year of the new vehicle being sold (Copeland et al. 2005). For example, car prices drop significantly in the fourth quarter of the year for the current year model because the next year’s model is likely to be already in the market. Finally, \( \beta_{0ij} \) captures unobserved effects of the make-model of the vehicle (we run both fixed and random effects specifications using a Hausman test to pick the preferred specification).
Testing Propositions Related to the Volume of Used Good Trade: Proposition 2 implies \( \frac{\partial VOT}{\partial s} < 0 \), while Proposition 4 implies \( \frac{\partial VOT}{\partial v} > 0 \).

We test both these propositions with the following specification for used vehicle \( j \) in year \( t \):

\[
VOT_{jt} = \gamma_0j + \gamma_1Reliability_{jt} + \gamma_2Deterioration_{jt} + sX_t + \epsilon_{jit}, \tag{24}
\]

where \( VOT_{jt} \) is the volume of trade for used vehicle \( j \) in year \( t \); the other variables have been defined previously. While we discuss details of the VOT measure later, we should point out that we do not actually observe the VOT directly in the data. We construct it by taking the ratio of the total number of transactions of a particular make-model to the total available stock of that particular make-model in the sample.

Proposition 2, which relates the reliability of the used good to the VOT, would be supported by a positive \( \gamma_1 \). Similarly, Proposition 4 relating deterioration to the VOT would be supported by a positive \( \gamma_2 \). In addition to these hypothesized effects, we include the vector of year dummies, \( X_t \), to control for year-specific shocks that impact the auto industry. Note that the household characteristics vector, the car characteristics vector, and the quarter dummies are not included because these variables are defined only at the level of an individual transaction, while the VOT specification is aggregated across households. Finally, \( \gamma_0j \) captures unobserved effects of the make-model of the vehicle.

5.3. Data and Measures

5.3.1. Data Sources. We assembled information from three different sources to construct our sample of observations. These are (i) the Consumer Expenditure Survey (CES), conducted by the Bureau of Labor Statistics; (ii) the Kelly Blue Book (KBB), compiled by a private industry source; and (iii) Consumer Reports magazine. We describe each of these sources below.

The Bureau of Labor Statistics maintains the CES panel to construct the Consumer Price Index. The CES is a very comprehensive survey that is implemented as follows. Each quarter, detailed interviews are conducted with 4,500 randomly selected households to gather information about their expenditures covering a large number of areas. Each quarter, 25% of households are dropped from the sample and replaced with new members. We use CES data from 1999 to 2002 for our analysis, confining our attention to the section on automobile purchases.

Each CES observation in the automobile ownership section of the survey includes information about the brand, model year, purchase year and month, and various household demographic variables. The make-model of any vehicle traded-in is also recorded, along with its odometer reading and other trim-level details.

The CES is the source for most of the variables used in our study.

Our second source of data is the annual issues of KBB, which is a syndicated data source that assembles price information for various models and vintages of automobiles derived from auctions and other dealer-dealer transactions. It is widely used in the auto industry to provide benchmark resale prices. We use this source to compute our deterioration measure, as well as parts of the net price paid measure.

Our third source of data is annual issues of Consumer Reports, which is a well-respected magazine that collects and publishes a variety of performance and reliability information on a variety of consumer products. We use this source to construct our vehicle reliability measure.

5.3.2. Measures. We discuss below the construction of the measures of the variables used in Equations (20) and (24). Table 5 summarizes the descriptive statistics for our sample.

Net Price Paid: Our price numbers come from the CES. Specifically, we use the Cash Paid by the consumer toward the purchase of the vehicle, which is recorded in CES. This number is the final dollar check that the consumer writes and is net of all applicable rebates, incentives, and allowances, including the trade-in allowance. This point is made even clearer if we look at the wording of the relevant question asked in the CES: “What was the amount paid for it after trade-in allowance and discount?”

Now, for purchases made without trade-ins, Cash Paid is the net price. However, constructing the corresponding measure for transactions with a trade-in is more complicated. The economic value of a new car being purchased with a trade-in is the cash paid plus the market value of the used good that has been traded in. It is imperative, therefore, to add the market value of the used good being traded in to the Cash Paid number from CES. The CES includes a “trade-in allowance,” but this number is an accounting entity recorded during the transaction, and there are several reasons to suspect that it deviates from the resale price for the traded in vehicle. As such, we construct our net price measure for trade-in transactions by combining the CES cash paid and the “Private Party Value” shown in the KBB data for the make-model and vintage of the traded in vehicle. KBB describes this number as the price a consumer can expect to get in a resale transaction if she sells in the open market.

In summary, when trade-ins are involved, the net price is given by Cash Paid (from CES) + Private Party Value of the traded in vehicle (from KBB).

Trade-In: A dummy variable (= 1) for transactions that involved a trade-in is recorded from the CES data.
Understanding the Role of Trade-Ins in Durable Goods Markets

Reliability: This is a reverse-coded measure of s. Following Desai and Purohit (1999), we record the reliability rating for make-model j in year t from Consumer Reports magazine. The rating is on a 1–5 scale, with a higher number indicating a more reliable vehicle (i.e., smaller s). Consumer Reports constructs this measure from reader surveys and its own in-house tests.\(^{20}\)

Deterioration: In the analytical model, deterioration is given by the ratio of the per period quality of a new vehicle (r) to its expected per period quality as a used vehicle (normalized to 1). The construct embodies deterioration along all relevant quality dimensions, including functional and aesthetic quality, so we seek a comprehensive measure. To accomplish this, we capitalize on the fact that the per period quality of a durable good in equilibrium is given by its implicit rental price. In other words, if a durable good is purchased at price \(p\), used for one period and sold at the end of that period at a price \(p_u\), then \((p - p_u)\) is directly related to the per-period quality of the good during the period of consumption (ignoring discounting). Following Porter and Sattler (1999), we construct our measure of the deterioration rate for a make-model from the pattern of prices as \((p_jt - p_jt) / (p_{jt} - p_{jt})\), where \(p_{jt}\) is the price of vehicle \(j\) aged \(k\) years at time \(t\). The numerator is the quality of a “newer” vehicle (aged 1–4 years), and the denominator is the quality of an “older” vehicle (aged 4–7 years). The higher this number, the greater the rate of deterioration.\(^{21}\)

Volume of Used Good Trade (VOT): We construct our measure (VOT) with the following data from the CES:

\[
VOT_{jt} = \frac{(Used\ Purchases)_{jt} + (Used\ Sales)_{jt} + (Used\ Trade-Ins)_{jt}}{(Total\ Stock)_{jt}},
\]

where \(j\) is a particular make-model and \(t\) is the year of observation. Each of the numbers above is

\(^{20}\)A possible concern is that the Reliability measure is ordinal, although we use it in our specification as a continuous measure. To address this concern, we replaced the ordinal measure with dummy variables and reran the regression. We find that our results are similar to the ones obtained with the continuous specification.

\(^{21}\)We chose vintages 1, 4, and 7, because cars newer than one year or older than seven years are rarely traded in.
computed as the aggregate of the relevant household observations recorded in the CES. Specifically, the numerator is the total number of transactions recorded for make-model $j$ in year $t$, including all purchases, sales, and trade-ins. The denominator is the total available stock of make-model $j$ recorded in the CES data in year $t$, which normalizes the huge differences in the stocks of different makes-models. Our measure lies between 0 and 1, with a higher number representing a greater fraction of the stock being traded in a given year.

**Vehicle Characteristics (Z):** Vehicle characteristics recorded from the CES include dummy variables for (i) make-model, (ii) air conditioning, (iii) automatic transmission, (iv) front-wheel drive, and (v) sunroof. These are similar to those used in previous work (e.g., Goldberg 1996).

**Household Characteristics (B):** Household characteristics recorded from the CES include household income, female head of household, and minority head of household.

**Year Dummies (X):** The transaction year from the CES is used to construct a set of calendar year dummies.

**Quarter Dummies (Q):** It has been well documented that automobile prices show seasonal variations within the quarters of a year and the model year of the new vehicle being sold (Copeland et al. 2005). We use a set of seven quarter dummies to control for this; e.g., Q1P represents a dummy that takes the value 1 if the household purchased the previous year’s model in the first quarter of the current year.

### 5.4. Results

#### 5.4.1. Net Price Specification

**Specification Testing:** We ran four models in increasing order of complexity. Model 1 includes only the Reliability and Deterioration variables, Model 2 adds characteristics of the automobile, Model 3 adds year and quarter dummies, and Model 4 adds household characteristics. Table 6(a) reports the results. Each of these additions improves the fit to some extent; in what follows, we discuss the results from the most comprehensive specification, namely, Model 4. A Hausman specification test rejects the random effects for the make-model dummies in favor of fixed effects ($\chi^2(25) = 78.23, p < 0.001$), and hence we discuss the fixed-effects results below. In the interests of space, we do not report estimates for each of the make-model-specific fixed effects.

**Results:** The positive coefficient on the interaction term of the trade-in dummy and deterioration ($\beta_4 = 526.59, p < 0.05$) reflects the effect of reliability on the trade-in incentive. We can see the magnitude of this effect by comparing trade-in incentives for low versus high reliability makes-models. To do this, we insert values into Equation (23) to obtain the magnitude of the trade-in incentive as $[5,016.71 - 665.02 \times (1 \times \text{Reliability}) - 1,376.43 \times (1 \times \text{Deterioration})]$. At the sample average value of deterioration, this reduces to $[3,199.68 - 665.02 \times (1 \times \text{Reliability})]$. For a vehicle with low reliability (one unit below the sample average) and average deterioration, the estimated incentive is $1,889.51$, whereas for a vehicle with high reliability (one unit above the sample average), the equivalent incentive is only $14.23$. This suggests that even apart from any direct benefits because of reliability (i.e., higher reliability lets the producer charge a higher price for the new good), there are indirect profit-enhancing benefits, in that higher reliability lets the producer offer smaller trade-in incentives.

The positive coefficient on the interaction term of the trade-in dummy and deterioration ($\beta_4 = 1,376.43, p < 0.1$) supports Proposition 3; i.e., an increase in the deterioration rate reduces the size of the trade-in incentive. The magnitude of this effect can be determined by inserting values into Equation (23). We can see that for makes-models with a greater deterioration rate (a standard deviation above the mean), the incentive shrinks to $387.78$, whereas it expands to $1,516.45$ for makes-models with a smaller deterioration rate (a standard deviation below the mean).

#### 5.4.2. Volume of Used Good Trade Specification

**Specification Testing:** For the VOT regression, we estimate the specification reported in Equation (25). Table 6(b) reports the results. Recall that this specification is aggregated over households and is at the make-model level. The Hausman test to assess a random effect versus a fixed effect specification for the make-model effects does not reject the null hypothesis that the random effects estimator is consistent ($\chi^2(5) = 4.80, p = 0.44$). Hence we report the random effects results.

**Results:** Proposition 2, which states that the volume of used good trade for a make-model increases in its reliability, is supported ($\gamma_1 = 0.0123, p < 0.01$). Similarly, Proposition 4, which states that the volume of used good trade for a make-model increases with the deterioration rate, is supported ($\gamma_2 = 0.0245, p < 0.1$).

To summarize, we obtain empirical results that are directionally consistent with our model predictions, with our model offering an internally coherent explanation for these empirical findings.

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22 An anonymous reviewer pointed out that VOT numbers should appropriately account for leasing, given the popularity of leasing in the 1990s and beyond. To address this point, we obtained information from a proprietary source (Lease Trak from CNW Research) on the percentage of cars leased for each make-model and each year in our CES sample. We included this variable in the VOT regression and find our results practically unchanged (to be more precise, we used three-year-lagged leasing percentages, because most car leases last for three years).
Table 6(a)  Net Price Estimates (Equation (20)) (Dependent Variable: Net Price for the New Purchase)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>21,558.95***</td>
<td>20,998.65***</td>
<td>20,490.39***</td>
<td>21,220.54***</td>
</tr>
<tr>
<td></td>
<td>(1,848.88)</td>
<td>(2,127.09)</td>
<td>(2,237.95)</td>
<td>(2,336.83)</td>
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<td>(2,027.63)</td>
<td>(2,039.50)</td>
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<td>−638.14</td>
<td>−623.30</td>
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<td></td>
<td>(447.90)</td>
<td>(429.21)</td>
<td>(449.22)</td>
<td>(450.02)</td>
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<td>Deterioration (β₃)</td>
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<td>−491.65</td>
<td>−526.59</td>
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<td>(431.54)</td>
<td>(431.54)</td>
<td>(438.36)</td>
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<td>594.51**</td>
<td>670.39**</td>
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<td>(304.83)</td>
<td>(323.23)</td>
<td>(323.50)</td>
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<tr>
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<td>(512.49)</td>
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<td>757.23*</td>
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<td>(420.53)</td>
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<td>699.51</td>
<td>729.51*</td>
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<tr>
<td>Adjusted $R^2$</td>
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<td>0.58</td>
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<td>0.60</td>
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<tr>
<td>$F$-value</td>
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<td>24.99***</td>
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<td>17.14***</td>
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<td>Number of observations</td>
<td>490</td>
<td>490</td>
<td>490</td>
<td>490</td>
</tr>
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</table>

Notes. Fixed-effects regression with make-model dummies. Hausman test: $\chi^2(25) = 78.23, p < 0.001$.

*p < 0.1; **p < 0.05; ***p < 0.01.
Table 6(b) VOT Regression (Equation (25)) (Dependent Variable: VOT)

<table>
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<tr>
<th>Variable (γk)</th>
<th>Estimate</th>
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<td>Intercept</td>
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<td>(0.0217)</td>
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<tr>
<td>Reliability</td>
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<td>(0.0040)</td>
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<td>Deterioration</td>
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<td>(0.0138)</td>
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<tr>
<td>Yr1999</td>
<td>−0.0214***</td>
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<tr>
<td></td>
<td>(0.0085)</td>
</tr>
<tr>
<td>Yr2000</td>
<td>−0.0012</td>
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<tr>
<td></td>
<td>(0.0085)</td>
</tr>
<tr>
<td>Yr2001</td>
<td>−0.0060</td>
</tr>
<tr>
<td></td>
<td>(0.0080)</td>
</tr>
<tr>
<td>R²-overall</td>
<td>0.3380</td>
</tr>
<tr>
<td>Wald χ²(5)</td>
<td>20.0600***</td>
</tr>
</tbody>
</table>

Notes. Random-effects regression; Hausman test: χ²(5) = 4.80, p = 0.44.
* p < 0.1; ** p < 0.05; *** p < 0.01.

6. Conclusions

This paper sought to examine the motivations of a producer of new durable goods to accept trade-ins as partial payment. In particular, we examine markets where durables deteriorate over time and where the extant owners of used goods know their quality levels better than prospective buyers in the resale market (the lemon problem).

Past explanations for trade-ins have turned primarily on cannibalization and commitment issues. Trade-in programs operate differently in our work. Offering an incentive for the trade-in transaction induces some owners to turn in their goods rather than to hold them. In effect, this increases the average quality of used goods offered for resale, which softens the negative impact of the lemon problem. In fact, in the absence of adverse selection trade-ins play no efficiency-enhancing role in our model.

From a broader theory standpoint, our results speak to the adverse selection issue in durable goods markets. Unlike other “burning money”-type mechanisms addressing adverse selection, such as warranties and brand names, our work shows that trade-in programs are uniquely useful in attacking this problem in durable goods markets. They increase resale market efficiencies, as witnessed by the increase in the volume of used good trade as well as an increase in the average quality of used goods in the market.

We hasten to add that despite the enhanced efficiency of resale transactions, trade-ins are a mixed blessing because the prices of new goods are increased for those consumers who do not trade in. At its core, a trade-in program involves price discrimination with ambiguous effects on overall welfare.

Our paper also speaks to managers about the introduction and proper design of trade-in programs. Our analysis shows that producers in durable goods markets should consider trade-in programs as a matter of routine. Despite cannibalization concerns arising from a more active resale market, a producer’s profits will inevitably rise from introducing trade-ins, given pervasive lemon problems.

We close with a discussion of some of the limitations of this research. First, incorporating trade-ins into a model where a durable asset lasts more than two periods would provide for richer substitution patterns among vintages of various ages. Second, our model considers a monopolist producer—future research could model producer competition to examine the strategic role of trade-ins. Third, consumers display endowment effects and related framing phenomena when they act as sellers. Embedding a richer model of consumers alongside our profit-maximizing producer, and perhaps testing the implications using controlled lab experiments, could yield insights into the market-level consequences of these behavior patterns. Fourth, our model is primarily applicable to situations where technology is fairly static. This limits its generalizability to situations where technological change may itself be a reason for offering trade-ins. Although our empirical model controls for the effect of technological change (through the “quarter dummies”), it would be useful to model this analytically and see if any fresh insights emerge.

Acknowledgments

This paper is based on Essay 1 of the first author’s doctoral dissertation at the University of Minnesota. The authors would like to thank Tony Cui, Ganesh Iyer, Duncan Simester, and participants at the SICS conference in Berkeley, California for helpful comments.

Appendix

A. Proof of Lemma 1. Refer to the payoff matrix outlined in Table 2 of the paper. There are a total of 25 strategies to be considered for a consumer across any two periods. Of these, we can trivially rule out the eight that are marked blank in Table 2 because a necessary condition for holding on to a peach or lemon in the current period is that a consumer should have purchased a new good in the previous period. In the following, we illustrate how to rule out some other nonoptimal consumer strategies. All the 12 nonoptimal strategies can be ruled out with similar arguments.

Strategy: \( n^{t-1}(\theta)h(\theta) \) (Buy new and hold on to a lemon)

The payoff in the current period for following this strategy for consumer \( \theta \) is \((1 - s)\theta \). However, no rational consumer would follow this because holding onto a lemon is dominated by a strategy wherein the consumer could sell off the lemon in the used market and buy a used good, obtaining a (expected) payoff of \( w\theta \). As long as some new good buyers sell their high-quality realizations, this strategy strictly dominates; i.e., \( w\theta > (1 - s)\theta \) since \( w > (1 - s) \).
Similar argument rules out \( h_{t-1}(\theta) n_t(\theta) \) (Hold a lemon and buy new).

Strategy. \( n^{t-1}(\theta) u'(\theta) \) (Buy new and buy used)

The payoff in the current period for following this strategy for consumer \( \theta \) is  \( p_u + (w \theta - p_s) = w \theta \). If buying the used good is an optimal action in the current period, then  \( w \theta > v \theta - p + p_s \). This implies that the consumer should never have purchased the new good in the previous period. A similar argument rules out \( u^{t-1}(\theta)n_t(\theta) \) (Buy used and buy new).

Now, we show why \( n^{t-1}(\theta) h_{t-1}(\theta) \) and \( h^{t-1}(\theta) n_t(\theta) \) cannot be ruled out.

Strategies. \( n^{t-1}(\theta) h_{t-1}(\theta) \) and \( h^{t-1}(\theta) n_t(\theta) \) (Buy new and hold a realization of peach)

The payoff from holding onto a peach is given by \((1 + s)\theta > v \theta - p + p_s\), which implies that \((1 + s)\theta - p_s > v \theta - p\).

It might be an optimal strategy for some consumers to consume a peach every period,\(^{23}\) but adverse selection prohibits the possibility of such a purchase in the used market. Note that buying a new good provides consumers with an option to consume a peach, which is closer to their “ideal” product. Also notice that selling a peach might not be optimal for these consumers because the peach would fetch the used good price, \( p_u \), which reflects the expected quality of used goods in the marketplace, \( w \). This expected quality, of course, is strictly less than the quality of a peach, \((1+s)\).

Turning to the ordering of the segments, we obtain a standard result in vertical differentiation models; i.e., \( 0 = \theta_2 \leq \theta_1 = \theta_1 \leq 1 \). This follows directly from incentive compatibility—the highest types (\( \theta_1, 1 \)) consume, on average, the highest average quality through a new good purchase every period, followed by (\( \theta_2, \theta_1 \)), who buy new and hold onto a peach that gives an intermediate level of quality. Next in the quality ladder are (\( \theta_3, \theta_2 \)), who buy a used good every period, and finally the lowest types (\( 0, \theta_3 \)), who do not make any purchase. Q.E.D.

A.1. Lemma TA1. In stationary equilibrium, \( 1/(1 + \alpha) \) of segment 2 consumers (potential holders) buy new goods every period while \( \alpha/(1 + \alpha) \) hold.

Proof. Let \( h_t \) be the proportion of segment 2 in the market for new durables in any period \( t \). This implies that \((1 - h_t)\) is the proportion that holds. Now, \( a h_t \) of these new good buyers would realize high used-good quality in period \( t + 1 \) and continue to hold. On the other hand, \((1 - \alpha)\) of these new good buyers would have a low-quality realization and be back in the market to buy new goods. However, the size of holders must remain constant across time because the market is in a steady state. In other words, we must have

\[
1 - h_t = ah_t.
\]

Because the distribution of consumer types is continuous, it must be true that there exists a set of consumers between those who consume new every period and used every period that prefer consuming a peach every period.

That immediately gives

\[
h_t = 1/(1 + \alpha).
\]

Hence, note that in stationary equilibrium in every period, \( 1/(1 + \alpha) \) of the strategic holders are in the market for a new good and \( \alpha/(1 + \alpha) \) hold. Of the new good buyers, \( \alpha/(1 + \alpha) \) are in the market because they held a peach in the previous period while \((1 - \alpha)/(1 + \alpha) \) are in the market because they realized a lemon in the current period.

For, \( \alpha = (1/2) \), this immediately implies that \( h_t = 2/3 \). Q.E.D.

B. Proof of Proposition 1. The trade-in incentive is given by

\[
p - p_1 = \frac{2s(1 + s - \nu)(2s^2 - 4(1 + \nu)(9 + \nu) + s(37 + 5\nu))}{(4 + 5s - 4\nu)(s^2 + 8s(3 + \nu) - 8(1 + \nu)(3 + \nu))},
\]

(A1)

Differentiating this equation with respect to \( s \) yields

\[
\begin{align*}
(210s^6 + 128(-1 + \nu)^4(3 + \nu)(9 + \nu) + 16s^5(31 + 9\nu) \\
- 64s(-1 + \nu)^3(3 + \nu)(73 + 9\nu) + s^4(5,767 + (1,232 - 279\nu)\nu) \\
- 24s^3(-1 + \nu)(719 + 9\nu(280 + 9\nu)) \\
+ 8s^2(-1 + \nu)^2(2,817 + \nu(1,252 + 99\nu)))
\end{align*}
\]

\((14 + 5s - 4\nu)(s^2 + 8s(3 + \nu) - 8(-1 + \nu)(3 + \nu))^2)^{-1}.

Note that the denominator is positive. Some algebraic manipulation shows that the numerator is also positive for \( s \in (0, 1) \) and \( \nu > (1 + s) \). Q.E.D.

C. Proof of Proposition 2. The VOT in the used good market is given by the ratio of the total number of used goods that change hands to the total stock of goods in any period. Because the size of the cheapskates segment is equal to the total number of used good purchases (which equals used good sales), we can write VOT as

\[
\text{VOT} = (\theta_2 - \theta_3)/2y,
\]

where the numerator is the size of the cheapskates segment and the denominator is the total stock of goods in any period. Substituting the values from Table 2 gives

\[
\text{VOT} = (1/2)\left[1 - \frac{s}{4(\nu - 1 - s)}\right].
\]

(A3)

Differentiating the RHS of (A3) with respect to \( s \) gives

\[
\frac{-8s}{(8 + 8s - 8s^2 - 1)} = \frac{1}{8(\nu - 1 - s)}.
\]

This is negative. Q.E.D.

D. Proof of Proposition 3. Differentiating (A1) with respect to \( \nu \) yields

\[
\begin{align*}
(2s(-57s^3 - 768(-1 + \nu)^4 + 128s(-1 + \nu)^2 \\
\cdot (-31 + 23\nu) + 2s^4(-532 + 99\nu) \\
+ 16s^3(-1 + \nu)(407 + 3\nu(-95 + 2\nu) \\
- 16s^2(270 + \nu(-221 + 15\nu))))
\end{align*}
\]

\((4 + 5s - 4\nu)(s^2 + 8s(3 + \nu) - 8(-1 + \nu)(3 + \nu))^2)^{-1}.

(A4)

Note that the denominator is positive and the numerator is negative for \( s \in (0, 1) \) and \( \nu > (1 + s) \). Q.E.D.
E. Proof of Proposition 4. Differentiating the RHS of (A3) with respect to $v$ gives

$$\frac{8}{(8 + 8s - 8v)^2}.$$ 

This is positive. Q.E.D.

References


Bruce, N., P. Desai, R. Staelin. 2005. The better they are, the more they give: Trade promotions of consumer durables. J. Marketing Res. 42(1) 54–66.


